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Abstract. We numerically and theoretically investigate the behavior of a granular gas driven by asymmetric plates. The injection of energy in the dissipative system differs from one side to the opposite one. We prove that the dynamical clustering which is expected for such a system is affected by the asymmetry. As a consequence, the cluster position can be fully controlled. This property could lead to various applications in the handling of granular materials in low-gravity environment. Moreover, the dynamical cluster is characterized by natural oscillations which are also captured by a model. These oscillations are mainly related to the cluster size, thus providing an original way to probe the clustering behavior.

1 Introduction

For high enough filling fraction, driven granular materials can exhibit a collective behavior known as dynamical clustering. Given the dissipative character of the collisions, kinematic energy is lost at each particle interaction. This loss leads to the formation of slow and dense regions within the granular medium [1, 2]. However, energy is also continuously injected into the system and rapidly a steady state is reached. The latter phenomenon was studied experimentally in microgravity [3–7] and rationalized theoretically and numerically [8–14].

The dynamical clustering can be explained considering the competition between a characteristic dissipation time, called Haff time, and a characteristic time of energy propagation through the system [14–16]. Since, gravity would induce another characteristic time in the system and affect dynamical clustering [17], low-gravity condition is needed for the study of this phenomenon. This motivates the European Space Agency’s (ESA) VIPGRAN project [18] in which granular gas will be experimentally investigated under various conditions on the International Space Station (ISS). Numerical work is essential to prepare the VIPGRAN project and to fix the experimental parameters.

The behavior and the stability of a granular cluster is poorly understood when the excitation parameters are changed. The main motivation of this article is to address the question of variable injection of energy in the system. The first step is to study the case of an asymmetric driving and to study the position of the cluster in the system. In the second step, the possible motion of the cluster will be analyzed. We will see in this paper that it is possible to control both position and motion of dynamical cluster, opening ways to achieved a granular transport in microgravity.

2 Numerical approach

Our numerical study relies on intensive Molecular Dynamic (MD) simulations, using a linear spring-dashpot model [14, 20, 21]. At each collision, normal and tangent forces are evaluated, respectively, through particle deformations and tangential velocities. The value of the coefficient of restitution $\varepsilon$ and the spring stiffness $k_n$ are both constant and fixed at 0.9 (corresponding to bronze beads) and $5 \cdot 10^5 \text{ N/m}$, respectively. A complete description of MD simulation is given by Taberlet in [22]. Moreover, the algorithm was already used by the authors in [13,14,20,21,23]. Based on the VIPGRAN concept [18], we designed a box with fixed lateral walls and moving top and bottom boundaries, modeling the pistons. The width and depth of the container are equal to $l = 15 \text{ mm}$. The length $L(t) = z_h(t) - z_c(t)$ is variable and depends on the positions of the oscillating plates according to following equations of motion:

$$
\begin{cases}
z_h(t) = z_h^0 + A_h \sin(2\pi ft), \\
z_c(t) = z_c^0 + A_c \sin(2\pi ft + \varphi).
\end{cases}
$$

The pistons are oscillating with a fixed frequency $f = 5 \text{ Hz}$ and a phase shift $\varphi$ as proposed in the VIPGRAN project. The average distance between the plates is $L = 50 \text{ mm}$. To ensure clustering in the system [14], the cell is filled with $N = 2000$ grains of radius fixed at $r = 0.5 \text{ mm}$. The amplitudes $A_h$ and $A_c$, defining the amplitude ratio $a = A_c/A_h$, and the phase shift $\varphi$ have been modified. We choose to keep the frequencies fixed at $5 \text{ Hz}$ for both plates in order to change the intensity of the injected energies while keeping a constant time scale in the system from one simulation to another. In addition, the residual vibration of the instrument is minimized if the frequencies are the same for both walls. This fact is important in order to reproduce the
Fig. 1. Sketch of the VIPGRAN cell used in simulations. Its dimensions are \( L = 50 \) mm and \( l = 15 \) mm. The oscillating plates have the following parameters: a common frequency fixed at \( f = 5 \) Hz and tunable amplitudes, \( A_c \) and \( A_h \). The length \( L \) is measured between plate equilibrium positions. The relevant parameter is the amplitude ratio \( a = A_c/A_h \).

Table 1. View of the variable parameters of the four runs of simulations. (Symbols correspond to those of figs. 5 and 4.)

<table>
<thead>
<tr>
<th>Run</th>
<th>( A_h ) (mm)</th>
<th>( a )</th>
<th>( \varphi )</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>[0; 1]</td>
<td>( \pi )</td>
<td>⋄</td>
</tr>
<tr>
<td>2</td>
<td>7.5</td>
<td>[0.2; 0.8]</td>
<td>( \pi )</td>
<td>■</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>[0.2; 0.8]</td>
<td>( \pi )</td>
<td>▲</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>1</td>
<td>[0; 2( \pi )]</td>
<td>▼</td>
</tr>
</tbody>
</table>

Simulated experiment in the VIPGRAN project. A sketch of the described cell is shown on fig. 1. Four campaigns of simulations have been realized. In the first three, we investigated the impact of the amplitude ratio while the fourth was devoted to the influence of the phase shift. We choose to fix the amplitude of the hot plate (the bottom one on fig. 1) at \( A_h = 5 \) mm, 7.5 mm and 10 mm in the first, second and third campaign, respectively. The tunable parameter of the experiment was the amplitude ratio \( a \) and was varied in the interval \( \{0; \frac{1}{4}; \frac{1}{2}; \ldots; 1\} \) and in the interval \( \{0.2; \ldots; 0.8\} \) in second and third runs. In the last ones, the amplitudes of both vibrating plates were set to \( A_h = A_c = 5 \) mm while the phase shift \( \varphi \) was varied in the interval \( \{0; \frac{\pi}{4}; \ldots; \frac{3\pi}{4}\} \). Table 1 gives a simplified view of the parameters used in the four campaigns. Each run corresponds to 300 periods of plates oscillation, requiring about \( 10^8 \) iterations of the algorithm.

Fig. 2. Three snapshots of the simulated experience for amplitude ratios \( a = 0.25 \) (a), \( a = 0.5 \) (b) and \( a = 1 \) (c). The cluster’s position is directed by the value of \( a \). The “hot” plate is always the bottom one and has an oscillation amplitude \( A_h = 5 \) mm corresponding to the first campaign. (d) Semi-log plot of the packing fraction of the grains contained in the cell along the \( z \)-axis for \( a = 0.5 \). The energy injection is made from \( t/T = 0 \) to \( t/T = 1/4 \). In this phase, both hot and cold gases are compressed by the pistons. The packing fraction for \( 0 < z/L < 0.2 \) (corresponding approximately to the localization of the cluster) remains very stable over the whole period \( T \).

3 Results

Figures 2(a), (b) and (c) present the granular system in the cell for different amplitude ratios \( a \). As remarked previously [13, 24, 25], we observe the formation of a dense region gathering together approximatively 95\% of the grains in the box. This low energetic region is separated from the pistons by two dilute gases of particles which transmit energy from the oscillating plates toward the cluster. One observes that the position of the cluster is a function of the amplitude ratio. Indeed, the smaller the amplitude ra-
tion is, the closer the cluster is to the “cold” plate \( (i.e.\) the one with the smallest oscillation amplitude). In addition, the packing fraction \( \eta \) of the system for \( a = 0.5 \) is shown in semi-log scale on fig. 2(d) along the \( z \)-axis. One can note that the cluster remains very stable over a whole period \( T = 1/f \). However, the packing fraction of both hot and cold gases are fluctuating with the position of the oscillating pistons taken at different times between \( t = 0 \) and \( t = T \). A second observation we made is the long-time oscillation of the cluster in the box. The position of the agglomerated grains is indeed not stable over multiple periods but makes a periodic motion with a pulsation very different from the plate’s ones. These oscillations are shown on fig. 3 for three different amplitude ratios, corresponding to the cases (a), (b) and (c) of fig. 2.

The condensation behavior of the grains was already studied and theoretically characterized \([14]\). The new feature of driven that is investigated in this paper is the control of the cluster’s position by amplitude tuning. This position, denoted by \( z_{cl} \), corresponds to the center of mass of the system. We observed that this quantity corresponds in good approximation to the densest region encountered in the cluster. For convenience, the \( z \)-axis is calculated from the center of the cell \( (z = 0) \) toward the cold plate.

Our simulations give evidence for an original behavior of driven granular media submitted to asymmetrically constraints in microgravity: a cluster is formed and is able to move in the cell and oscillates about its equilibrium position. Although the cluster is oscillating during long simulations, an average position, noted \( \langle z \rangle \), can be calculated and linked to the amplitude ratio. Figure 4 gives this normalized average position for the first three campaigns. Note that the phase shift between both plates has no influence on the equilibrium position of the cluster.

4 Cluster equilibrium position

To model the equilibrium position process in a simple way, we consider the cluster like a dense and stable pile of condensed grains receiving two momentums coming from both hot and cold plates. The model is based on two fundamental hypotheses: i) the cluster has reached equilibrium and does not move anymore and ii) the momentums coming from both hot and cold plates are sent after each period to the cluster, with the help of gaseous grains. Two conditions coming out of these hypotheses are also found. The momentums sent by the vibrating walls have to be equal and the time needed for each momentums to cross the cell has to be the same. Assuming that the number of particles contained in the cluster is constant, \( i.e.\) that the cluster captures as many grains as it loses through evaporation, we can model the cluster’s equilibrium position. As stated in previous works \([8, 24, 26, 27]\), the distribution of the velocities in both hot and cold gases can be normalized by the maximal velocities of their respective pistons. This scaling leads to identical distributions for both hot and cold gases and supports our hypothesis that a grain coming from the plate \( i = \{h, c\} \) has a typical velocity

\[
v_i = A_i \omega. \tag{2}\]

Moreover, we consider that the grains coming from the plates impact the cluster with unchanged velocities. This hypothesis has been confirmed by measuring gas densities. The mean free path we calculated was indeed roughly \( l \approx 80 \text{ mm} \). In comparison with the length of the cell \( L \), this value implies that a grain coming from a vibrating plate will probably not collide with another grain before it attains the cluster. As discussed above, the equilibrium is reached when the time needed to reach the cluster is equal for both plates. Considering a straight and uniform movement of the grains, this time is directly linked to the plate-cluster distance by \( \Delta t_{cl} = \Delta z_{cl}/v_i \). Equaling \( \Delta t_h \) and \( \Delta t_c \), we find the following equilibrium condition:

\[
\frac{\Delta z_c}{\Delta z_h} = \frac{A_c}{A_h}. \tag{3}\]

Taking into account the cluster’s thickness \( e \) along the \( z \)-axis, we can develop \( \Delta z_{cl} \) in (3) in order to link it to the cluster’s position \( z_{cl} \). These distances are illustrated on the sketch of fig. 1. Using eq. (3) we find the cluster’s equilibrium position

\[
z_{cl} = \frac{L - e}{2} \left( \frac{1 - a}{1 + a} \right), \tag{4}\]

where the dependence of the amplitude ratio \( a \) is obtained. The next step consists in finding the cluster’s thickness \( e \). For this purpose, we used an algorithm that measures the local density \( \eta \) around each sphere in the cell. Previous work \([14]\) has shown that the clusters have an approximate local density larger than 0.285. Using this criterion, we defined the cluster’s thickness by the maximal vertical distance that separates two particles with a local density.
\( \eta \geq 0.285 \). Measurements were made on the entire simulation and averaged to finally find \( e \approx 10 \text{ mm} \), whatever the value of \( a \). This result is not surprising since the excitation given by the oscillating plates is so intense that the cluster cannot be more compressed, even for the first campaign, making its density fairly constant. As a consequence, the cluster’s thickness is stable since its definition is directly linked to the local density of the grains. In fig. 4, we have plotted eq. (4) with a fixed value of \( e = 10 \text{ mm} \). The model is in excellent agreement with the results found in our numerical campaigns.

5 Cluster oscillations

As discussed before and illustrated in fig. 3(b), clusters are able to oscillate in the box around their equilibrium position \( z_{e}^{\ast} \). The oscillations originate from a Hopf bifurcation as proposed previously in \([25,28]\). Note that the amplitude of these oscillations is significative in regards to the value of the cluster thickness \( e \). The cluster’s inertia is given by the total mass \( M_{c} = N_{c}m_{c} \), where \( N_{c} \) is the number of grains of mass \( m \) contained in the cluster. According to our observations, this number of particles is roughly constant. In addition, we consider that the cluster interacts with the moving planes via both hot and cold gases that contain, respectively, \( N_{h} \) and \( N_{c} \) particles with \( N_{h} \) not necessarily equal to \( N_{c} \). These numbers of particles are linked together by the total number of grains injected in the box \( N = N_{h} + N_{c} + N_{h} \). However, within our strongly dilute granular gases (see fig. 3), only a part \( n_{h} < N_{h} \) and \( n_{c} < N_{c} \) of the particles are close enough to the vibrating walls to collide with them. These grains carry momentum towards the cluster. One has

\[
\begin{align*}
\Delta p_{h} &= n_{h}mA_{h}\omega, \\
\Delta p_{c} &= n_{c}mA_{c}\omega.
\end{align*}
\]

Considering that the cluster’s position is perturbed by \( \delta \), we can find an equation giving the momentum rate received by the cluster as a function of \( \delta \). Momentum rates out of equilibrium can be decomposed into two parts corresponding to the oscillating plates:

\[
\begin{align*}
\frac{\Delta p_{h}}{\Delta t_{h}} &= \frac{n_{h}(z_{c}^{\ast} + \delta)m(A_{h}\omega)^{2}}{(L - e)(1 + a) + \delta}, \\
\frac{\Delta p_{c}}{\Delta t_{c}} &= \frac{n_{c}(z_{c}^{\ast} + \delta)m(A_{c}\omega)^{2}}{(L - e)a(1 + a) - \delta}.
\end{align*}
\]

In order to solve eq. (6), one has to evaluate the number of grains carrying momentums from the hot and the cold plates. Assuming that both hot and cold gases are uniformly distributed along the \( z \)-axis, \( n_{h}(z_{c}^{\ast} + \delta) \) and \( n_{c}(z_{c}^{\ast} + \delta) \) are given by

\[
\begin{align*}
n_{h}(z_{c}^{\ast} + \delta) &= \frac{N_{h}A_{h}}{(L - e)(1 + a) + \delta}, \\
n_{c}(z_{c}^{\ast} + \delta) &= \frac{N_{c}A_{c}}{(L - e)a(1 + a) - \delta}.
\end{align*}
\]

From eqs. (6) and (7), the total force that acts on the cluster as a function of the perturbation \( \delta \) is defined as

\[
F(z_{c}^{\ast} + \delta) = \frac{N_{h}A_{c}^{2}m\omega^{2}}{((L - e)(1 + a) + \delta)^{2}} - \frac{N_{c}A_{h}^{2}m\omega^{2}}{((L - e)a(1 + a) - \delta)^{2}}.
\]
In order to find a theoretical value for the cluster’s oscillation frequency, we linearized the force \( F(z_{cl}^* + \delta) \) around the point \( z_{cl}^* \). This linearization leads finally to

\[
F(z_{cl}^* + \delta) \approx -2(1 + a)^3 A_h^3 m(N_h + N_c) \omega^2 \delta. \tag{9}
\]

The equation giving the evolution of the perturbation \( \delta \) is then reduced to a harmonic oscillator with a typical pulsation

\[
\omega_{cl} = \left( \frac{A_c + A_h}{L - e} \right)^{3/2} \left( \frac{2}{3} \frac{N - N_{cl}}{N_{cl}} \right)^{1/2} \omega. \tag{10}
\]

This pulsation is only function of a single free parameter which is the number of grains contained in the cluster. As a consequence, eq. (10) gives the natural frequency of the cluster even for symmetric energy injection. Moreover, the mass of the cluster could be evaluated by measuring this pulsation. Measures of \( \omega_{cl} \) for different amplitude ratios were performed with a Fast Fourier Transform (FFT) algorithm and plotted on fig. 5. The data are fitted with the free fitting parameter \( N_{cl} \) using eq. (10) and are in good agreement with the model except for small values of \( a \) for which the oscillations of the cluster are too weak to be detected by the algorithm. A remarkable feature is that the fitting number of particles in the cluster was found to be equal to \( N_{cl} \approx 1860 \) for all campaigns. This result means that the dilute gases are composed of approximatively 140 grains, as observed in many simulations.

As seen from eqs. (4) and (10), the phase shift between the hot and cold plates should have neither influence on the equilibrium position nor on the pulsation of the cluster. These results are expected because the models are both based on a momentum balance over one period of oscillation. If the driving amplitudes and frequencies are fixed, the average momentums transmitted to the system remain the same regardless of the phase shift. The fourth campaign that was devoted to the phase shift influence has confirmed this fact.

Finally, notice that such oscillations of granular materials have been recently reproduced numerically under gravity by Rivas et al. [28]. The authors have performed simulations of a vibrated granular medium composed of particles confined in a quasi-one-dimensional system. In this case, low-frequency oscillations of the media have been observed. If the behaviors are similar, some differences have to be highlighted. First, Rivas et al. have been able to describe theoretically their observations with a continuum model. In our work, the granular gases observed are so dilute that this way is not relevant. Secondly, the authors have seen that when more energy is injected, the frequency of cluster oscillations becomes smaller. We observed exactly the inverse. The simplest explanation is found in the nature of the movements observed. In the gravitational open environment of the authors, the energy given to the cluster determines its parabolic flight, which increases with the initial energy of the body. The time of flight of the cluster increases also with the initial energy injected. It is not the case in our closed system under microgravity where the cluster adopts a straight and uniform motion. The time to cross a determined distance is then so small that the kinetic energy of the moving cluster is intense. Surprisingly, the existence of a periodic collective motion is not only observed under gravity field but also in microgravity environment, although the nature of these motions are different.

6 Application to granular transport

The above results suggest that it is possible to create and control grain displacements in microgravity. In order to prove this concept, we propose a system inspired by Maxwell’s demon [23,29] and granular ratchets [30–33] for generating granular transport in low-gravity environment. Several cells with independent pistons are placed in a row. Specific apertures at different heights allow a granular exchange between neighboring cells. By controlling the amplitude ratio of the independent pistons it is possible to drive the cluster in a selected cell. Figure 6
shows a simulation of a directed-grains experiment. Three boxes are placed together and connected with two slits placed at different heights. The simulation starts with all the grains in the central cell (fig. 6a). The amplitude condition in the central cell drives the cluster in front of the first slit, while in the first cell the amplitude ratio is inverted in order to drive the incoming cluster at the bottom of the cell. After a while, most of the grains are trapped at the bottom of the first cell (fig. 6b). Then, the amplitude ratios are inverted in all cells in order to drive the cluster from the left to the right cell. Note that no intermediate cluster is observed in the central cell (fig. 6c) but that the grains directly gather in the right cell (fig. 6d). We performed different simulations with different numbers of cells and similar behaviors have been found. By inverting the amplitude ratios, the granular transport is reversible.

7 Conclusion

In this paper, a study of the behavior of a dynamical cluster of grains excited by an asymmetric constraint was performed with the help of Molecular Dynamics. A model was developed in order to link both cluster’s position and cluster’s oscillations to both hot and cold amplitudes. The natural frequency of a dynamical cluster has been emphasized and could be used to estimate the mass of the cluster. The model provides a way to produce granular transport as checked in our simulations.

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